Basic Math Review

Basic Math Review

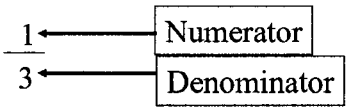
Outline

- Fractions
- Multiplying Fractions
- Dividing Fractions
- Basic Math Operations
- Exponents
- Logarithms

Fractions

- Defined: A ratio of whole numbers and is one or more of the equal or unequal portions of anything
- Example

$$\frac{1}{3} = 0.33$$



Multiply Fractions

- Multiply another whole number (called an integer) with the numerator.
- Examples

$$\frac{1}{3} \times 5 = \frac{(1 \times 5)}{3} = \frac{5}{3}$$

$$\frac{12}{16} \times 14 = \frac{(12 \times 14)}{16} = \frac{168}{16}$$

Dividing Fractions

- Multiply the integer by the reciprocal of the fraction
- Examples:

Reciprocal of $\frac{4}{5} \longrightarrow \frac{5}{4}$

Reciprocal of $\frac{16}{10} \longrightarrow \frac{10}{16}$

$$\frac{10}{\frac{4}{5}} = 10 \times \frac{5}{4} = \frac{50}{4} = 12.5 \qquad \frac{24}{\frac{23}{15}} = 24 \times \frac{15}{23} = \frac{360}{23} = 15.7$$

Basic Math Operations

- Very powerful when dealing with complicated conversions.
- Rule: If the numerator and denominator of a fraction are multiplied or divided by the same number, the value of the fraction is not changed

$$\frac{3}{4} = \left[\frac{3}{4}\right]\left[\frac{2}{2}\right] = \frac{6}{8}$$

- Rule: If numerator and denominator of a fraction are added or subtracted by the same number, the value of the fraction is changed.

$$\frac{3}{4} \neq \frac{3+2}{4+2} = \frac{5}{6}$$

Basic Math Operations

- Multiplication of a fraction by a whole number

$$5 = \frac{5}{1} \longrightarrow 5\left[\frac{15}{23}\right] = \frac{(5 \times 15)}{23} = \frac{75}{23} = 3.3$$

- Solving unknowns

(1) Addition/Subtraction: $a + b = b + a$

Means that you must change both sides of an equation by same value in order to maintain equal value on both sides

$5 + x = 18$, Solve for value of x

Rule: Add or subtract equal value on both sides of equation. to get your known values on one side of the equation to determine value of unknown.

(1) Subtract the value of 5 on both sides of equation

$5 - 5 + x = 18 - 5$

$0 + x = 13$

$x = 13$

Reason: Mathematically to get all known values on one side of the equation.

Basic Math Operations

- Solving unknown with fractions

$$\frac{a}{b} = \frac{a}{b}$$

Rule: To solve an unknown denominator or numerator, get known values on one side of the equation.

$$\frac{X}{15} = \frac{23}{5}$$

$$(15)\frac{X}{15} = \frac{23}{5}(15)$$

Mathematically get "15" to the side with the other known values by multiplying "15" on both sides

$$(1)X = \frac{23}{5}(15)$$

$$X = 69$$

Basic Math Operations

- Solving unknown with fractions

$$\frac{a}{b} = \frac{a}{b}$$

Rule: To solve an unknown denominator or numerator, get known values on one side of the equation.

$\frac{15}{X} = \frac{23}{5}$ — Get all known values on one-side of equation; get unknown to be a numerator.

$5\left(\frac{15}{X}\right) = \left(\frac{23}{5}\right)5$ — Here, multiply both sides by "5"

$X\left(\frac{75}{X}\right) = (23)X$ — Here, multiply both sides by "X" to get "X" to one side as a numerator.

$\left(\frac{1}{23}\right)75 = X(23)\left(\frac{1}{23}\right)$ — Here, divide "23" into both sides of equation to get "23" mathematically on the same side as "75."

$\underline{\mathbf{3.3}} = X$

Basic Math Operations

- Solving unknown

$a(b+c) = a(c+b)$

Rule: To solve an unknown denominator or numerator, get known values on one side of the equation.

$23(x+12) = 155$ — Get all known values on one-side of equation; get unknown to be a numerator.

$23x + (23)(12) = 155$ — Here, multiply left side of equation

$23x + 276 - 276 = 155 - 276$ — Here, subtract both sides by "276" to get "276" on right side of equation.

$\frac{23x}{23} = \frac{-121}{23}$ — Here, divide "23" into both sides of equation to get "23" mathematically on the same side as "-121."

$x = \underline{\mathbf{-5.3}}$

Basic Math Operations

- Solving unknown

$\frac{a(b+c)}{d} = \frac{a(c+b)}{d}$

Rule: To solve an unknown denominator or numerator, get known values on one side of the equation.

$\frac{15\,(34+x)}{45} = 598$

$\frac{15\,(34+x)}{45}(45) = 598(45)$ Multiply both sides with "45."

$(15)(34) + (15)(x) = 598(45)$ Multiply the numbers on left side of equation.

$15x = 598(45) - (15)(34)$
$x = 1760$

Here, divide "23" into both sides of equation to get "23" mathematically on the same side as "-121."

Exponents

- A mathematical expression to represent the number of times an integer is multiplied to itself.

$5 \times 5 \times 5 \times 5 \times 5 = (5)^5$ $(4 \times 4 \times 4) + 5 = (4)^3 + 5$

- When a negative sign (-) is placed on an exponent, it represents the inverse of the integer.

$(5)^{-5} = \frac{1}{3125}$ $(4)^{-3} = \frac{1}{64}$

- Rule of Exponents:
  - Exponent of 1 EQUALS the integer. $(5)^1 = 5$ $(3)^1 = 3$
  - Exponent of 0 EQUALS 1: $(5)^0 = 1$ $(3)^0 = 1$

Exponents

• Fractions with exponents: Apply exponent to both the numerator or denominator individually, or simply divide the numerator into the denominator & then, apply the exponent.

$$\left[\frac{3}{4}\right]^4 = \frac{(3)^4}{(4)^4} = (0.75)^4 \qquad \left[\frac{15}{23}\right]^{14} = \frac{(15)^{14}}{(23)^{14}} = (0.652)^{14}$$

• Fractional Exponents:
  • Determine the dividend from the fractional exponent and use on the whole number.
  OR
  • Use a logarithm

$(5)^{1/2} = (5)^{(0.5)} = \mathbf{2.2}$

Logarithm

• A form of exponential math expressions.

• Two Types
  • Logarithms (Log): The power to which a base of 10 must be raised to obtain a number. (*LOG* on Calculator)

  • Natural Logarithms (ln): The power to which the base e (e = 2.718281828.....) must be raised to obtain a number (*LN* on Calculator)

Log Examples:

• Logs are EXPONENTS.

$\log_{10}100 = 2$ is equivalent to $10^2 = 100$

$\log_{10} 5 = 0.7$ is equivalent to $(10)^{0.7} = 5$

Logarithms

• Natural Logarithm Example: (Base is e = 2.718281828 or simply 2.718)

$\ln 30 = 3.4012 = e^{3.4012}$ **OR** $2.7183^{3.4012} = 30$

$\ln 15 = 2.71 = e^{2.71} = 15$ **OR** $2.7183^{2.71} = 15$

• Logarithms Involving Fractions: Either log (or ln)the dividend or subtract log of denominator from log of numerator.

$\text{Log}\left[\frac{15}{23}\right] = \text{Log}(15) - \text{Log}(23) = \mathbf{-0.1856}$

$\text{Log}\left[\frac{123}{44}\right] = \text{Log}(123) - \text{Log}(44) = \mathbf{0.446}$

$\ln\left[\frac{15}{23}\right] = \ln(15) - \ln(23) = \mathbf{-0.43}$

$\ln\left[\frac{123}{44}\right] = \ln(123) - \ln(44) = \mathbf{1.03}$

Logarithms

• Solve problems involving fractions:

$\text{Log}_{10}\left[\frac{X}{15}\right] = 452$

$\text{Log}_{10}(X) - \text{Log}_{10}(15) = 3$ — Separate out the fraction into a subtraction equation. Solve the log of the known value (e.g., "15").

$\text{Log}_{10}(X) - 1.2 + (1.2) = 3 + 1.2$ — Add 1.2 (=Log (15) from both sides in order to move your known values on one side of equation.

$\text{Log}_{10}(X) = 4.2$

$10^{(4.2)} = X = \mathbf{15849}$ — Remember in previous slide the alternate expression of this equation

(Press "4.2" on calculator, then press "2nd" key, then press "10^x" key.)

ln (natural logs) are done the exact same way BUT different key function and value.

Logarithms

• Solving Other Problems:

$x(\text{Log } 20) + 2 = 13$

$x(1.30) + 2 = 13$ — Determine the Log of 20 (*Press "20" & "LOG" key*)

$x(1.30) + 2 - 2 = 13 - 2$ — Subtract "2" from both sides to get known values on one side.

$\frac{x(1.30)}{1.30} = \frac{11}{1.30}$ — Divide "1.30" from both sides to get 1.30 on one side. "1.30" divided by "1.30" is equal to "1."

$x = 8.5$

$5(15 - \text{Log } x) + 23 = 11$

$75 - 5(\text{Log } x) + 23 = 11$ — Multiply out the products in the parenthesis.

$(75+23) - 5(\text{Log } x) = 11$ — Add known values on left side of equation.

$98 - 98 - 5(\log x) = 11 - 98$ — Subtract 98 from both sides to get 98 to move on right side of equation.

$\frac{-5(\log x)}{5} = \frac{-(11 - 98)}{5}$ — Divide 5 from both sides to get 5 on right side with the other known values.

$\log x = 17.4 = 10^{(17.4)} =$ — Re-express the log x into an exponent to solve. (*On calculator, press "17.4"), "2nd" Key, then "10ˣ".*

2.5×10^{17}

Root Functions

• Square Root - A math expression representing an exponent of a number to (1/2).

$x^{(1/2)} = \sqrt{x}$

$2^{(1/2)} = \sqrt{2} = \mathbf{1.41}$ (*Calculator, Press "2", then "$\sqrt{x}$ "*)

• Other Types of Root Functions

$x^{(1/3)} = \sqrt[3]{x}$ Called a Cube Root

$x^{(2/3.5)} = \sqrt[3.5]{x^2}$

$\left[\frac{x}{y}\right]^{\frac{2}{3}} = \sqrt[3]{\left[\frac{x}{y}\right]^2}$

Root Functions

$\sqrt{145} =$ Press "145" and then, the "$\sqrt{x}$" key.

$\sqrt{[25\,(x+4)]} = 15$

$\sqrt{(25x+100)} = 15$ Multiply products inside parenthesis

$\left[\sqrt{(25x+100)}\right]^2 = [15]^2$ Square both sides of the equation to cancel out the square root.

How does that happen?

Remember that $x^{(1/2)} = \sqrt{x}$

If square a square root, the value of the exponent becomes "1."

$x^{(1/2)\,x\,2} = x^{(1)}$

$25\,(x+4)^2 = 15$

$\frac{25\,(x+4)^2}{25} = \frac{15}{25}$ Divide both sides by 25 to get 25 on otherside.

$\sqrt{(x+4)^2} = \sqrt{\left[\frac{15}{25}\right]}$ To remove the square, $(\ )^2$, is to Square Root both sides.

$x + 4 - 4 = 0.77 - 4$ Subtract 4 from both sides to get 4 on right side.

x = - 3.3

Group Exercise

Using calculator, solve for x in the following problems

1. $\frac{2}{5} + x = 42$

2. $20\,\text{Log}\left[\frac{x}{12}\right] = 12$

3. $12 + \frac{(12 - x)}{42} = \text{Log } 24$

4. $\ln\left[\frac{12}{x}\right] - 42 = 12$

Group Exercise

Using calculator, solve for x in the following problems

5. $14 \ln\left[\frac{x}{12}\right] = 5$

6. $\ln\left[\frac{(14 + x)}{24}\right] + 13 = 11$

7. $26 \ln\left[\frac{23}{(16 - x)}\right] - 45 = 9$

Group Exercise

Using calculator, solve for x in the following problems

8. $\sqrt{(24 + x)} = 36$

9. $\left[\frac{23}{x}\right]^5 + 25 = 14$

10. $\frac{\log(\sqrt{(x + 5)^3})}{23} = 73$

Practice Test: Basic Math

5 Questions
Work Individually
5 minutes

1.0 Battle Drills:
Basic Math Review

1. What is the value of the following?

$$\frac{\frac{15}{14}}{\frac{23}{16}}$$

a. 0.98
b. 0.54
c. 2.4
d. 0.75

2. Solve the following x:

$(23)(15+x) = 157$

a. –11.7
b. 11.17
c. -8.17
d. 1.23

3. $\log\left[\frac{(10+x)^2}{243}\right] = 2$

a. –32.4
b. 2433.22
c. –12.3
d. 146.7

4. $\left[\sqrt{(12-x)^{2.5}}\right]\left[(\ln(12-3)^3\right] = 32$

a. 8.46
b. 8.20
c. 9.34
d. 10.2

5. $\left[\frac{\sqrt{(45-x)^5}}{75}\right] = 17$

a. 12.4
b. 32.2
c. 27.5
d. 11.0